Part 1: 50 points, please describe your answers as complete as possible.

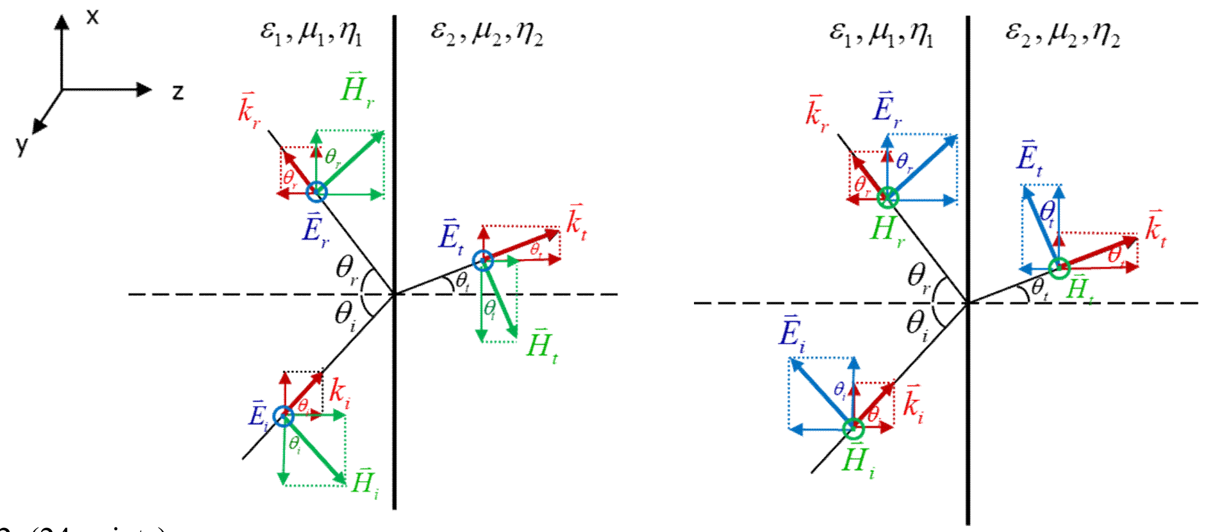
1. (10 points)
   A circular loop of N turns of conducting wires lies in the xy-plane with its center at the origin of a magnetic field specified by $\vec{B} = \hat{a}_z B_0 \cos(\pi r/2b) \sin\omega t$, where b is the radius of the loop and ω is the angular frequency. Please find the emf induced in the loop.
2. (40 points)
   (a) Please write down the Maxwell equations (4 equations) with free charge ρ_f and free current density $\vec{J}_f$, and describe their physical significant. (8 points)
   (b) Please derive four boundary conditions for electromagnetic fields based on the Maxwell equations. (20 points)
   (c) Please derive the wave equation for electric and magnetic fields in a simple-medium (source-free, lossless). You may need the vector identify $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$ (12 points)

Part 2: 50 points, please describe your answers as complete as possible.

1. (16 points)

From Maxwell's Equations, please derive the **reflection coefficients** of oblique angle incident light for (a) perpendicular polarization (b) parallel polarization. Here the convention $e^{j\omega t - j\vec{k}\cdot\vec{r}}$ is used.

$$k_i = \hat{x}\beta_1 \sin\theta_i + \hat{z}\beta_1 \cos\theta_i,\ k_r = \hat{x}\beta_1 \sin\theta_r - \hat{z}\beta_1 \cos\theta_r,\ k_t = \hat{x}\beta_2 \sin\theta_t + \hat{z}\beta_2 \cos\theta_t$$



2. (24 points)

A uniform plane wave propagating in dielectric medium 1 ($n_1 = \sqrt{2}+1 = \sqrt{\varepsilon_{1r}}$)with E field as

$$\vec{E}_i(x,z) = \vec{E}_{\|} + \vec{E}_{\perp} = (\cos\theta_1\hat{x} - \sin\theta_1\hat{z})C_1 e^{-j2\pi\sqrt{\varepsilon_{1r}}(\sin\theta_1 x + \cos\theta_1 z)} + \hat{y}C_2 e^{-j2\pi\sqrt{\varepsilon_{1r}}(\sin\theta_1 x + \cos\theta_1 z)+j\phi}$$

is obliquely incident upon the semi-infinite medium 2 ($n_2 = 1 = \sqrt{\varepsilon_{2r}}$) located at the xy plane at z=0.

Here $C_1=C_2=1$ and $\phi=\pi/2$ $\theta_1 = \pi/8$.

(a) (6 points) Find the **critical** angle and the **Brewster** angle

(b) (6 points) Find the **polarization** sense of the **incident** wave and the **reflected** wave

(c) (6 points) Find the expression of transmitted wave **E_t** and its **polarization** sense

(d) (6 points) From(c), **decompose** the E_t as two circular polarized waves with amplitude C_1 for left hand circular polarization and C_2 for right hand circular polarization. Find C_1 and C_2=?

3. (10 points)

An atom-thick graphene layer with sheet conductivity $\sigma(z) = \sigma_0\delta(z)$ is located at z=*0*. For a *normal* incident plane wave propagating in ***z*** direction with ***E*** field polarized in ***x*** direction, find the **transmission coefficient *t*** and **reflection coefficient *r***=? (use methods similar to the derivation of the transmission/reflection coefficient of normal incident to an dielectric interface, except now there is a sheet current density $J_x(z) = \sigma_x(z)E_x(z) = \sigma_0\delta(z)E_x(z)$ in $j\omega\varepsilon_0 E_x + \sigma_0\delta(z)E_x = (\nabla\times H)_x = -\frac{\partial H_y}{\partial z}$

Part 3: 50 points, please describe your answers as complete as possible.

1. (35 points)

   For an infinite dielectric (made of simple medium) slab waveguide of thickness *d* situated in air, answer the following questions about the instantaneous expressions of all nonzero field components, eigenvalue equation, and cutoff frequency for **odd TE modes**. Remember to show your detailed steps.

   (a) In a source-free environment, rewrite the real-field Maxwell's equations into complex-field Maxwell's equations for a time-harmonic electromagnetic wave.

   (b) Derive the inter-relationships among all complex-field components according to (a).

   (c) Derive a wave equation for the complex electric field based upon (b).

   (d) Assume the electromagnetic wave propagates along the z-direction in Cartesian coordinates. Based on (c), derive the z-components of the complex electric field in all regions for odd TE modes, where the boundary condition, the tangential component of the electric field must be continuous across the boundary, is used.

   (e) Based on (b) and (d), derive other complex-field components in all regions for odd TE modes.

   (f) Based on (e), obtain the instantaneous expressions of all the field components for odd TE modes in all regions.

   (g) Based on (e), derive the eigenvalue equation for odd TE modes by using the boundary condition, the tangential component of the magnetic field must be continuous across the boundary for non-conducting medium.

   (h) Based on (g), derive the cutoff frequency for odd TE modes.

2. (15 points)

   An air-filled parallel-plate conducting waveguide has a plate separation of 1.25 cm.

   (a) Find the cutoff frequencies of TE_0, TM_0, TE_1, TM_1, and TM_2 modes.

   (b) Find the phase velocities of the above modes at 15 GHz.

   (c) Find the group velocities of the above modes at 10 GHz.

   (d) Find the lowest-order TE and TM mode that cannot propagate in this waveguide at 25 GHz.