Section 1

1. Express the electric field $E_x(z,t) = 50\cos(10^7 t - 0.6z + 40°)$ V/m in a phasor form. (5 pt).

2. For a uniform plane wave with complex amplitude, $\mathrm{E} = 50\hat{x} + 30\angle 40°\hat{y}$ V/m, please express the phasor and real instantaneous fields if the wave propagates in the +z direction in the free space and has frequency of 200 MHz. (15 pt).

3. The wave number k, which can be related to attenuation and phase constant by $\mathrm{jk} = \alpha + \mathrm{j}\beta$. Please express α and β respectively in terms of real and imaginary parts of the complex permittivity (10 pt).

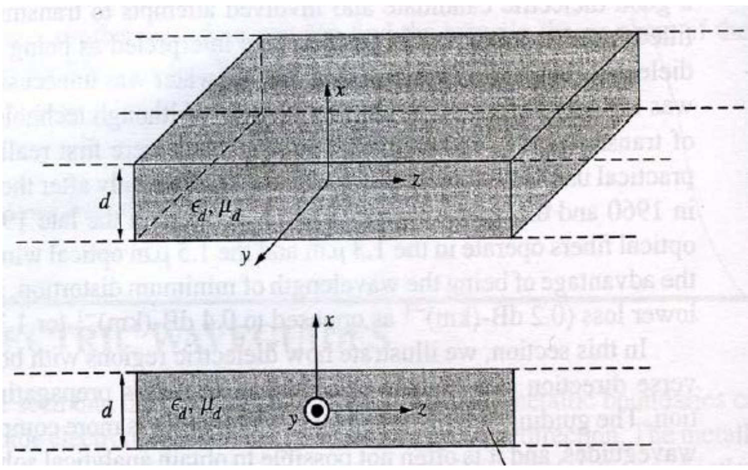
4. The power density of the laser pointer is on the order of 1 mW. Assume the output wave is a uniform plane wave $E_x = E_{x0}\cos(\omega t - \beta z)$. Please (A) Derive the corresponding time-average power density and (B) Calculate E_{x0} (20 pt)

Section 2

5. The effective complex dielectric constants of walls in buildings are investigated for wireless communication applications. The relative dielectric constant of the reinforced concrete wall of a building is found to be $\varepsilon_r = 6.7 - j1.2$ at 900 MHz and $\varepsilon_r = 6.2 - j0.69$ at 1.8 GHz, respectively. (a) Find the appropriate thickness of the concrete wall to cause a 10 dB attenuation in the filed strength of the 900 MHz signal traveling over its thickness. Neglect the reflections from the surfaces of the wall. (10 points) (b) Repeat the same calculations at 1.8 GHz (10 points)
6. Derive that the attenuation constant of TE waves between parallel metal plates is given by

$$\alpha_{CTEm} = \frac{2R_S\left(f_{C_m}/f\right)^2}{\eta a\sqrt{1-\left(f_{C_m}/f\right)^2}}$$

   where RS and f_{C_m} are the resistive part of the surface impedance and cutoff frequency of the TE waves. (20 points)
7. Find the field solutions and cutoff frequency for TM waves within a dielectric waveguide as shown in following figure. (10 points)



Section 3.

8. (30%) Hertz dipole antenna

the *time varying Maxwell equations* in differential form with current **J** and charge density ρ

are $\nabla \cdot B = 0$ (1) $\nabla \times E = -\frac{\partial B}{\partial t}$ (2) $\nabla \times B = \mu\varepsilon \frac{\partial E}{\partial t} + \mu J$ (3) $\nabla \cdot E = \frac{\rho}{\varepsilon}$ (4)

(a) From Eq. (1) the magnetic B field can be expressed in terms of a vector potential A, and from Eq.(2) the electric E field can be expressed in term of vector potential A and scalar potential V. Please write down B and E fields in terms of A and V

(b) From Eq.(3) and Eq. (4) derive the non-homogeneous *wave equations* for vector potential **A** and scalar potential V with current **J** and charge density ρ using the Lorentz *gauge* ($\nabla \cdot A + \mu\varepsilon \frac{\partial V}{\partial t} = 0$)

(c) The solutions of **A** and V in integral form are

$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int_{v'} \frac{\vec{J}(\vec{r}',t-R/c)}{R} dv'$ and $V(\vec{r},t) = \frac{1}{4\pi\varepsilon}\int_{v'} \frac{\rho(\vec{r}',t-R/c)}{R} dv'$

where r' is the position of the source, $R = |\vec{r} - \vec{r}'|$, and c is the speed of light.

For time harmonic source $\vec{J}(\vec{r}')e^{i\omega t}$, the solution is $\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int_{v'} \frac{\vec{J}(\vec{r}')e^{i\omega t - i\beta R}}{R} dv'$ where $\beta = \omega/c$. Thus $\vec{A}(\vec{r},\omega) = \frac{\mu}{4\pi}\int_{v'} \frac{\vec{J}(\vec{r}')e^{-i\beta R}}{R} dv'$. Now consider a Hertz dipole with length of ***dl<<λ***, R→ r, driven by current ***I*** in the ***z*** direction. Find A_r , A_θ , A_ϕ =?

(d) From (c) and use $\nabla \times A = \frac{\hat{r}}{r\sin\theta}\left(\frac{\partial(A_\phi \sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right) + \frac{\hat{\theta}}{r\sin\theta}\left(\frac{\partial A_r}{\partial\phi} - \sin\theta\frac{\partial(rA_\phi)}{\partial r}\right) + \frac{\hat{\phi}}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right)$,

derive the ***H*** fields from **A**

(e) Find the electric field **E**

(f) Three elementary antenna are linearly spaced λ/2 apart and are excited in phase with amplitude ratio 1:2:1. Find the far field electric field, and sketch its radiation pattern.

9. (20%) Consider a noble metal such as gold or silver with free electrons driven by an electric field $E = E_0 e^{i\omega t}$ and the electrons are damped with damping rate γ due to collision loss. Suppose a single electron with charge of q_e and mass of m_e, and the damping rate is γ.

(a) Please write down the force equation of a driven damped free electron in noble metal

(b) Let $x = x_0 e^{i\omega t}$, substitute into the above expression and find x_0 in terms of $q_e, E_0, m_e, \omega, \gamma$

(c) From the definition $D = \varepsilon_{eff} E = \varepsilon_0 E + P$ and $P = N_e q_e x_0$ where N_e is the volume density of electrons, find the <u>effective permittivity</u> $\varepsilon_{eff}(\omega)$ in terms of γ ε_0 and $\omega_p^2 = \frac{N_e q_e^2}{m_e \varepsilon_0}$ (plasma frequency).

(d) Find the <u>real part and imaginary part</u> of $\varepsilon_{eff}(\omega)$ and draw it schematically.