1. (10 points) Numerical Aperture of an Optical Fiber

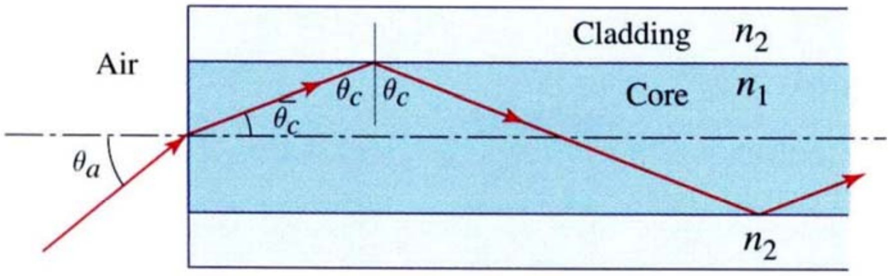


Figure 1.2-18 Acceptance angle of an optical fiber.

Please derive $\mathrm{NA} = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$

2. (10 points) Please derive Snell' s law. (Hint: using "the wavefronts of the two waves match" , or the Fermat' s Principle.)

3. (20 points) The Half-wave Retarder. Linearly light is transmitted through a half-wave retarder. If the polarization plane makes an angle with the fast axis of the retarder, show that the transmitted light is nearly polarized at an angle -θ. i.e., rotates by an angle 2θ. Why is the half-wave retarder not equivalent to a polarization rotator?

4. (10 points) Electric Field of Focused Light. (a) 1 W of optical power is focused uniformly on a flat target of size 0.1 x 0.1 mm^2 placed in free space. Determine the peak value of the electric field E_0 (V/m), Assume that the optical wave is approximated as a transverse electromagnetic (TEM) plane wave within the area of the target. (b) Determine the electric field at the center of a Gaussian beam (a point on the beam axis at the beam waist) if the beam power is 1 W and the beam waist radius W_0 = 0.1 mm.

1. (10 points) (Resonator Optics) The frequency spacing between adjacent resonator modes: $\nu_F = \frac{c}{2d}$, and the spectral width of the individual resonator modes: $\delta\nu \approx \frac{\nu_F}{\mathcal{F}}$. The round-trip intensity attenuation factor ($|r|^2 = \mathcal{R}_1\mathcal{R}_2\exp(-2\alpha_s d)$) can be written as $|r|^2 = \exp(-2\alpha_r d)$. Please show the effective loss coefficient (α_r) using $\mathcal{R}_1$, $\mathcal{R}_2$, and α_s.

2. (10 points) (Radiation Process) Please plot the radiation processes of the spontaneous emission, stimulated emission, and absorption.

3. (10 points) (Laser Mechanism) Please explain the working principles of "Q-Switching" and "Mode-Locking".

4. (20 points) (Electro-Optic Effect in $LiNbO_3$)

The $LiNbO_3$ is a uniaxial crystal with ordinary and extraordinary refractive indexes (n_0 and n_e). The applied electric field is parallel to the z axis. The electro-optic tensor of $LiNbO_3$ in the form of

$$r_{ij} = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}.$$

(4-a) (10 points) Please derive the birefringence of $n_z - n_y$ when the propagation of light along the x axis.

(4-b) (10 points) After propagation a distance L, the two modes undergo a relative phase retardation given by $\Gamma = k_0[n_z - n_y]L = k_0\left[(n_e - n_o) - \frac{1}{2}(n_e^3 r_{33} - n_o^3 r_{13})E(n_1 - n_2)\right]L$. If E is obtained by applying a voltage be V between two surfaces of the medium that are separated by a distance d, please derive the "Retardation Half-Wave Voltage".

1. Consider an InGaAsP-InP laser diode which has an optical cavity of length 250 microns. The peak radiation is at 1550 nm and the refractive index of InGaAsP is 4. The optical gain bandwidth (as measured between half intensity points) will normally depend on the pumping current (diode current) but for this problem assume that it is 2 nm.

(a) What is the mode integer *m* of the peak radiation?(5 points)

(b) What is the separation between the modes of the cavity? (5 points)

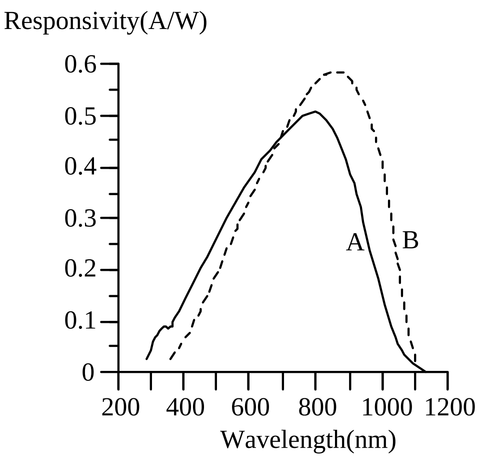
(c) How many modes are there in the cavity? (10 points)

2. Consider two commercial Si *pin* photodiodes, type A and type B, both classified as fast *pin* photodiodes. They have the responsivity shown in following figure. Differences in the responsivity are due to the *pin* device structure. The photosensitive area is 0.125 cm^2 (4 mm in diameter).

(a)Calculate the photocurrent from each when they are illuminated with blue light of wavelength 450nm and light intensity 1 μW cm^{-2}. What is the QE of each device? (10 points)

(b) Will the A or B photodetector be more sensitive to the radiation from a GaAs laser? Why?(5 points)

(c) A photodetector whose area is 5×10^{-2} cm^2 is irradiated with yellow(λ=600 nm) light whose intensity is 2 mW cm^{-2}. Assuming that each photon generates one electron-hole pair, calculate the number of pairs generated per second. (5 points)



The responsivity of two commercial Si *pin* photodiodes

3. If the width of the relative light intensity vs. photon energy spectrum of an LED is typically around **~3k_BT**, what is the linewidth, $\triangle\lambda_{1/2}$, in the output spectrum in terms of wavelength?(a) GaAs LED emits at a peak wavelength of 870 nm(5 points) (b) InGaAsP LED emits at a peak wavelength of 1550 nm. (5 points)

note: $\dfrac{\Delta\lambda}{\Delta E_{ph}} \approx \left|\dfrac{d\lambda}{dE_{ph}}\right|$